EVALUATION OF RATES OF HEAT AND MASS TRANSFER

IN ARTIFICIALLY AGITATED FLOWS

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Heat and mass transfer in a turbulent flow is subjected to an examination in which the decay of turbulence close to solid surfaces is taken into account.

The method used in the present paper for theoretical evaluation of the rates of heat and mass transfer in artificially agitated flows is based on the use of the ideas of semiempirical turbulence theory [1, 2] and consideration of the decay of turbulence near solid surfaces [3]. Taking into account that mass and heat transfer is effected by molecular and turbulent transfer, we write an expression for the mass flow* from a mass-transfer surface

$$j_D = (D + D_T)\,\frac{dc}{dy}. \tag{1}$$

On the basis of Landau's hypothesis [3], which has been confirmed experimentally in recent years [4, 5], we assume that the decay of the coefficient of turbulent transfer in the viscous sublayer is proportional to the fourth power of the distance from the wall. In this case Eq. (1) can be written in the form

$$j_D = \left(D + \frac{v_0 y^4}{\delta_0^3}\right)\frac{dc}{dy}. \tag{2}$$

It has been shown in several studies [6, 7] that for media characterized by $\mathrm{Pr} > 1$, the viscous sublayer region provides the main diffusion resistance, which almost completely determines the magnitude of the diffusion flow. Taking this into account, we can write the following expression for the diffusion resistance of the boundary layer:

$$R = \int_0^\infty \frac{dy}{D + \dfrac{v_0 y^4}{\delta_0^3}} = \frac{\pi}{2\sqrt{2}D}\sqrt[4]{\frac{D\delta_0^3}{v_0}}. \tag{3}$$

From which the mass transfer coefficient is

$$\beta = \frac{1}{R} = 0.9D\sqrt[4]{\frac{v_0}{D\delta_0^3}}. \tag{4}$$

*An analogous expression can be written for the case of heat transfer from a heat-transfer surface.

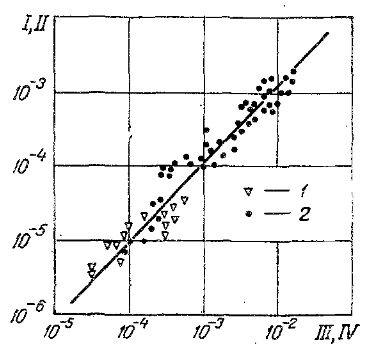

Fig. 1. Heat- and mass-transfer coefficients as function of mixing conditions in mixer apparatuses from results of measurements [11-13]. 1) Mass transfer; 2) heat transfer and from Eqs. (9) and (10) (continuous line): I) β; II) $[\alpha/c\rho]$, m/sec; III) $(\varepsilon_0\nu)^{0.25}/\mathrm{Pr}_T^{0.75}$; IV) $[(\varepsilon_0\nu)^{0.25}/\mathrm{Pr}_D^{0.75}]$, m/sec.

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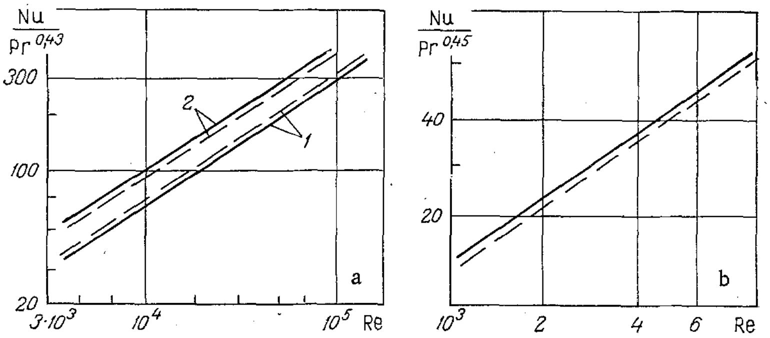

Fig. 2. Heat transfer during turbulent flow of liquid in round tubes containing agitating devices: 1) Helical agitator [14]; 2) blade-type agitator [14] (a); b) agitating mixer [15] [the dashed lines are the results of measurements, and the continuous lines are obtained by calculation from Eq. (10)].

The quantity δ_0 — the distance from the solid surface at which the effect of molecular viscosity begins to act — and the velocity v_0 are connected [8, 9] by the relation

$$\mathrm{Re}_{cr} = v_0\delta_0/\nu = 11.5. \tag{5}$$

Expressing δ_0 from this we have

$$\beta = 0.133 v_0 \mathrm{Pr}_D^{-0.75}. \tag{6}$$

The quantity v_0 is usually expressed as $v_0 = \sqrt{\tau/\rho}$ and in its physical sense can be regarded [8, 9] as the fluctuation velocity.

It follows from (5) that at a distance of δ_0 from the surface the turbulent viscosity is approximately an order greater than the molecular viscosity of the liquid and, hence, δ_0 — the scale of the fluctuations of velocity v_0 — greatly exceeds the internal turbulence scale λ_0, given by the relation [3]

$$v_{\lambda_0}\lambda_0/\nu \simeq 1. \tag{7}$$

Since the thickness of the viscous sublayer δ_0 is small in comparison with the turbulence macroscale we can regard the turbulence in an artificially agitated flow as locally isotropic and use the Kolmogorov—Obukhov "two-thirds law" [1, 2] for the evaluation of v_0:

$$v_0^2 \simeq (\varepsilon_0^{2/3}\delta_0^{2/3}). \tag{8}$$

Whence, taking (5) into account, we can write the expression (6) in the following way:

$$\beta \simeq 0.245(\varepsilon_0\nu)^{1/4}/\mathrm{Pr}_D^{3/4}. \tag{9}$$

It should be noted that an analogous relation between the mass-transfer coefficient and the complex $(\varepsilon_0\nu)$ was obtained in [10] on the basis of dimensional analysis, but the lack of physical premises prevented the author obtaining a final expression for evaluation of the mass-transfer coefficient.

A similar method can be used for analysis of heat transfer from a solid surface in a turbulent flow. In this case the expression for the heat-transfer coefficient takes the following form:

$$\alpha \simeq 0.245C\rho(\varepsilon_0\nu)^{1/4}/\mathrm{Pr}_T^{3/4}. \tag{10}$$

To test the obtained Eqs. (9) and (10) we used the results of experimental investigations of heat and mass transfer for fixed surfaces in mixer apparatuses [11-13] and tubes containing agitating devices [14, 15].

The dissipation of the energy of turbulent motion ε_0 was determined from data for the energy expenditure:

$$\varepsilon_0 = N/\rho V_a. \tag{11}$$

For mixer apparatuses the power was calculated by the usual method [16]. A comparison of the calculated values of the heat- and mass-transfer coefficients with the results of experimental measurements is shown in Fig. 1.

Since the power in mixer apparatuses is expressed as [16]

$$N = K_N \rho n^3 d_{\text{м}}^5, \tag{12}$$

we can convert Eqs. (9) and (11) to the form usually used for the treatment of experimental mixing data:

$$\mathrm{Nu} = 0.245 K_N^{0.25} \mathrm{Re}_{\mathrm{c}}^{0.75} \mathrm{Pr}^{0.25} (D_{\text{a}}/d_{\text{м}})^{0.25} (H_{\text{a}}/D_{\text{a}})^{0.25}. \tag{13}$$

The obtained Eq. (13) is practically the same with regard to the nature of the effect of the dimensionless variables as the empirical equations given in [16].

For tubes containing agitating devices, the power expended on overcoming the hydraulic resistance is:

$$N_{\text{т}} = \Delta P G = \lambda \frac{L}{d} \frac{\rho V^2}{2} \frac{\pi d^2}{4} V. \tag{14}$$

Then, in accordance with (11)

$$\varepsilon_0 = \frac{\lambda V^3}{2d}, \tag{15}$$

and the heat-transfer equation (10) takes the form

$$\mathrm{Nu}_{\text{т}} = 0.205 \mathrm{Re}^{0.75} \mathrm{Pr}_{\text{т}}^{0.25} \lambda^{0.25} \tag{16}$$

Figure 2a, b shows a comparison of the results of calculation from this equation with experimental values obtained from measurements of heat transfer in tubes containing agitating devices of different types [14, 15]. In the calculation we used the values of the resistance coefficient λ found in these investigations.

As Figs. 1 and 2 show, the results of calculation agree quite satisfactorily with the results of measurement. Additional confirmation of the obtained relations is provided by a comparison of the theoretical value of the index of the resistance coefficient λ in Eq. (16) with the experimentally determined value of 0.24 for a ribbed annular channel.

NOTATION

D, D_T, molecular and turbulent diffusion coefficients, m²/sec; δ_0, thickness of viscous sublayer, m; $v\lambda_0$, velocity of turbulent fluctuations of scale λ_0, m/sec; v_0, velocity of liquid at distance δ_0 from wall, m/sec; y, distance from wall, m; ε_0, energy dissipated per unit mass, W/kg; C, heat capacity, J/kg·deg; ρ, liquid density, kg/m³; V_a, volume of apparatus, m³; N, power expended on mixing, W; K_N, power coefficient; n, frequency of rotation of mixing device, sec⁻¹; d_m, diameter of mixer, m; D_a, diameter of apparatus, m; H_a, depth of liquid in apparatus, m; λ, hydraulic resistance of tube; V, mean velocity of liquid in tube, m/sec; $\mathrm{Re} = Vd_T/\nu$; $\mathrm{Re}_c = nd_M^2/\nu$.

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MULTICOMPONENT DIFFUSION IN LIQUID MIXTURES BY CHROMATOGRAPHIC AND NMR TECHNIQUES

UDC 536

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A method of calculating the matrix of the coefficients of multicomponent diffusion in liquid mixtures on the basis of experimental data obtained by chromatographic and pulsed nuclear magnetic resonance (NMR) techniques is devised.

Information about diffusion coefficients in liquid multicomponent mixtures is of theoretical and practical interest. Theoretical methods still do not provide accurate predictions of diffusion in liquids. Diffusion in gases and solids can be calculated fairly accurately, but in the case of liquids rigorous extimates are difficult, since the molecules in liquids are tightly packed and the dynamics of their interaction is multiparticulate.

The results of measurements of diffusion coefficients in liquid systems are sparse, since the widely used classical methods greatly increase the duration of the experiment. Attempts to devise express methods of investigating diffusion in mixtures have recently been made. They include chromatographic and pulsed NMR techniques, the theory and application of which in the investigation of diffusion have been confined so far only to binary gas and liquid mixtures, which are determined by one diffusion coefficient.

For multicomponent mixtures molecular mass transfer is written in the form

$$(\mathbf{J}) = -[D]\,\nabla(\mathbf{c}), \tag{1}$$

where [D] is the matrix of the multicomponent diffusion coefficients (MMDC).

It has been stressed in many recent papers [1, 2] that the values of the nondiagonal MMDC elements may attain values commensurable with the diagonal elements, and that their values depend strongly on the concentration and composition of the mixtures. Hence, there was a need to devise a simple, but accurate, method of obtaining the MMDC.

The chromatographic method consists in the creation of a concentration perturbation in a laminar flow by injection of a sample at the entrance to a long capillary tube followed

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